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MODELING MOMENTS OF ORDER THREE AND FOUR OF DISTRIBUTION OF YIELDS

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Abstract

The different ways of assessing risk are numerous in the financial literature. Regulation has made it a function of Value-at-Risk. Recalling the inadequacy of current approaches, the recent crisis is encouraging us to have tools giving more information on losses and gains. In recent years, modern statistics have developed a series of probabilistic tools that today have direct applications in finance. This is particularly the case for the use of the order three and four moments in the Valut-at-Risk calculation. For the study of leptokurticity, or non-Gaussianity, and the asymmetry of the distribution of yields. This work aims to apply this technique to types of financial assets including CAC40 member companies. The analysis of the regulation and its evolution following the economic crisis of 2008 make it possible to define the stakes and the difficulty of the measurement of the risks. The presentation of the concept of moments of order three and four exhibits their properties and shows that they bring more information on the tails of distributions than do the classical moments. The distribution of the returns of these securities makes it possible to show that this is a case of financial assets whose behavior is far from a normal distribution and thus requiring special techniques. Finally, the empirical analysis of CAC40 financial securities over a long period shows the benefit of order three and four moments in stalling laws and building more robust quantile estimators than those built from a law. Normal or by retaining historical distributions.

Keywords: Valut-at-Risk, the study of leptokurticity, and the asymmetry.

JEL Classification: G20, O11, O16,

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1. Introduction

The theory of finance concerns the study of the prices of financial products and more precisely their temporal evolutions. A financial model is most often based on a representation of the prices of financial assets (or interest rate levels). Such representation can be sought from the perspective of a better understanding of financial markets, or in a prospective development of tools to improve financial management: risk management, asset allocation, creation of new products financial.

Financial modeling always balances the adequacy of observations in financial markets with its convenience of use. Modeling that seeks to produce all the statistical properties of observations generally leads to a complicated model that often becomes difficult to use, theoretical calculations being generally difficult to conduct. On the other hand, excessive simplification of the models makes it possible to carry out numerous calculations, but at the expense of the efficiency of taking into account the structure of the returns. In this paper we simulate the Value-at-Risk based on the Monte Carlo simulation and then calculate the Gaussian Value-at-Risk with different levels of quantiles, and then in the end we move to Value-at-Risk according to approaches (Cornish Fisher, Gram Charlier, and Jonson) who take into consideration the three and four moments of the distribution. We analyze the main statistical properties of the financial series and we propose a coherent modeling of most of these properties.

So is the VaR (concept of ruin) used in insurance and then in the trading rooms (JP Morgan - RiskMetrics) she played a very important role in the analysis of risk and financial reserves. The sophistication of market instruments and their aggregation into more complex portfolios (arbitrage, hedging, multiple asset classes) has stimulated research to improve this tool. It has become indispensable for establishments with complex activities, and the regulator has made it a key element in loss and profile evaluation models. VaR has been criticized for its inadequacies (variety of results according to the laws used, information on the risk very different according to the level of confidence retained, non-additivity). The crisis also highlighted new or insufficiently formalized risks (liquidity, model risk, endogeneity, systemic risk). It also calls for a better understanding of the behavior of assets in the extremities (maximum loss or gain), for this techniques were developed from order statistics in order to build estimators that better take into account the maxima information in distributions. A recourse to the moments of order three and four proves useful for the calculation of a VaR which would contain more information on the realizations of the profits and losses.

Thus, starting from the standard approach of Bollerslev and Zhang (2003), we will include in these models of asset valuation higher order moments (moments of order three and four). Indeed, we will be inspired by the theoretical justifications of Jurczenko and Maillet (2006) to introduce in the calculation models of the VaR, the moments of orders three and four. To better exploit this information provided by the proposed extensions on classical models of valuation with higher order moments (see Jurczenko and Maillet, 2006), Several measures of risk assessment of an asset from homogeneous distributions proposed in the literature of (see Back and Weigend, 1997 and Giot and Laurent, 2004) will be used below to compare different approaches. These are the Gaussian VaR (characterized by the mean and standard deviation of the security's returns), the Gram-

Charlier VaR (the asset returns follow a GC distribution), the VaR-Jonson (asset returns follow distributions from Jonson) and VaR-Cornish-Fisher (asset returns follow Cornish Fisher distributions). Thus, to compare and analyze the reliability of the various proposed alternative VaR measures, several backtesting procedures will also be implemented on the basis of the most used tests. We will use the failure proportion test (Kupiec) which is based on the estimated number of VaR exceedances estimated by the model. We will also use the Christoffersen confidence interval test.

To complete our project we use two titles of the CAC40: Total and Bouygues, chosen for their characteristics different from the moments of order three and four. In the first part, we will study the stylized facts that make it possible to identify the problem of non-normality of autocorrelation, or of volatility. When the second part is devoted to the presentation of VaR. And finally the last part is the subject of a study where modeling (empirical applications), for a period ranging from 02/01/2008 to 28/12/2013.

2. Value-at-Risk and Modeling of the Three and Fourth Moments of Financial Returns Distributions

By definition, VaR is the maximum loss that a portfolio manager may experience during a certain period of time with a given probability. Assuming that this probability is 95%, the margin of error for this maximum loss is only 5% if the cash flow distribution of a portfolio obeys a normal distribution. Suppose also that the random variable X represents the value of the portfolio, with X ~ N (μ, σ). The random variable X can be rewritten in terms of standard normal variable ε centered reduced:

$$\Pr(r_h < \text{VaR}) = p \qquad (1)$$

With $r_h = \text{Ln}\left({V_{t+h}}/{V_t}\right)$ the yield of the asset on the horizon h, and V_t the value of the index at time t. By constriction this VaR is a generally negative number. The $\emptyset$ (P) is the quantile function of the normal centered reduced law, with the known value of p and h, the VaR can also be written:

$$\text{VaR} = \propto_h + \sigma_h \times \emptyset^{-1}(\text{P}) \qquad (2)$$

Where $\propto_h$ is the quantile level and σ_h is the standard deviation of yields on the horizon h, and $\emptyset^{-1}$(P) the quantile function of the reduced normal centered equation.
The value of the VaR reflects the amount of the loss that the investor could not exceed with a certain probability over a specific time horizon. This approximation does not take into account extreme events that may occur that could lead to more severe losses. This leads the investor to make biased decisions based on VaR by underestimating the losses that can be avoided by using other estimation methods.

The extreme events produce irregularities in the yields, but also breaks that are reflected in their distributions by asymmetry and leptocurticity. To account for these phenomena we retain in this study the Gram-Charlier, Cornish-Fisher and Johnson methods, which use the first four moments of returns, and provide the approximate quantities of the unknown distribution of a portfolio return. . Using the VaR formula, this quantity approximately $\emptyset^{-1}(P)$, the quantile function of the yield distribution.

2.1. Gram-Charlier

The formula of VaR in equation (1) requires the quantile function for the approximate Gram-Charlier density. Which can be inverted numerically to calculate the VaR. the Gram-Charlier approximation is given by:

$$\emptyset_{GC}(P; k_3, k_4) = \emptyset_N - \frac{k_3}{6}[f_N \times (k^2-1)] - \frac{(k_4-3)}{24}[f_N \times k(k^2-3)] \quad (3)$$

whether $\emptyset_{GC}(z; k_3, k_4) = \left[1 + \frac{k_3}{6}(z^3-3z) + \frac{(k_4-3)}{24}(z^4-6z^2+3)\right] \times f_N(z)$

Where $\emptyset_N$ and f_N are respectively the standard normal distribution and density function evaluated at k, $k_3 = E[z^3]$ is the skewness coefficient and $k_4 = E[z^4]$ is the kurtosis coefficient. The VaR of Gram-Charlier is then calculated as follows:

$$VaR_{GC} = \propto_h + \sigma_h \emptyset_{GC}^{-1}(z; k_3, k_4) \quad (4)$$

2.2. Cornish Fisher

The Cornish-Fisher approach (see, for example, Zangari [1996]) leads to the following approximation:

$$w_\propto = Z_\propto + \frac{Z_\propto^2 - 1}{6}k_3 + \frac{Z_\propto^3 - 3Z_\propto}{24}(k_4 - 3) - \frac{2Z_\propto^3 - 5Z_\propto}{36}{k_3}^2$$

Where w_α is the corrected percentile of the distribution at the α threshold, $Z_\propto = \Phi_N^{-1}(\propto)$ where α is the quantile level, $\Phi_N^{-1}(\propto)$ the quantile function of the normal distribution centered reduced, k_4 the coefficient of kurtosis and k_3 the coefficient of skewness. The Cornish-Fisher VaR is written as follows:

$$VaR_{CF,\propto} = \mu + \left(Z_\propto + \frac{Z_\propto^2 - 1}{6}k_3 + \frac{Z_\propto^3 - 3Z_\propto}{24}(k_4 - 3) - \frac{2Z_\propto^3 - 5Z_\propto}{36}{k_3}^2\right)\sigma$$

With μ the mean and σ the standard deviation of the distribution, so the Fisher VaR can write as follows:

$$VaR_{CF,\propto} = \mu - w_\propto \times \sigma$$

For skewness distributions below zero or negative and where the kurtosis is greater than three, the VaR is shifted to the left relative to the Gaussian VaR and thus allows to take into account the deviations from "normality". By hypothesis this VaR correctly represents the risk if k_3 is close to zero, and if k_4 is close to three. If these two conditions are not satisfied, the result of the Cornish-Fisher approximation will be less relevant. The computation of VaR with the Cornich Fisher approach requires the knowledge of $\Phi_N^{-1}(\propto)$ by a dichotomous procedure to obtain the probability close to the quantile value.

The approximate quantile functions generated by the Cornish-Fisher approach do not always have desirable properties. It does not always generate a monotonic function for all pairs of asymmetry and flattening. Outside this set, the Cornish-Fisher expansion provides non-monotone quantiles in the tail of the distributions.

2.3. Johnson approch

We present this approach of Johnson from Simonato's method (2010). This approach of Johnson is presented by Simonato (2010) allows to use the first four moments as main input within a model of VaR.

A continuous random variable z with an unknown distribution can be approximated by the methodology proposed by Johnson (1949) from a set of "normalized" translations. This transforms the continuous variable z into a normal standard variable y and has the following general form:

$$y = a + b \times g\left(\frac{z - c}{d}\right)$$

Where a and b are the shape parameters, c is a location parameter, d is a scale parameter and g (.) Does a function whose form defines the four families of the distributions constitute the Johnson system.

$$g(\mu) = \begin{cases} \text{Ln}(\mu) \\ \text{Ln}\left(\mu + \sqrt{\mu^2 + 1}\right) \\ \text{Ln}\left(\frac{\mu}{1-\mu}\right) \\ \mu \end{cases}$$

They correspond respectively to the log-normal family, the unbounded family, the bounded family, and the normal family.

Thus, the process of using the Johnson system comes down to identifying the values of a, b, c and d that are associated with the moments of the distribution.
Hill et al. (1976) propose an algorithm for choosing the appropriate family and the parameter values required to approximate this unknown distribution when we know the first four moments of the function.

Johnson's random variable can express from the inverse of the previous normalized translation:

$$z = c + d.\, g^{-1}\left(\frac{y-a}{b}\right),$$

$$\text{whither} \quad g^{-1}(\mu) = \begin{cases} e^{\mu} \\ (e^{\mu} - e^{-\mu})/2 \\ 1/(1 + e^{-\mu}) \\ \mu \end{cases}$$

Which in the order correspond to the log-normal family, the unbounded family, the bounded family, and the normal family.
The quantities required for the VaR formulas are obtained from the asymmetry coefficient and the flattening coefficient of the standardized yield distribution. Using centered and reduced parameters within the Hill algorithm, we find the values of the parameters a, b, c and d. Once this stage is completed, it is then possible to measure the risk of this distribution.

Digital applications and discussions

We illustrate the usefulness of taking into account the three and four moments in the calculation of VaR from the series of assets Total and Bouygues over a daily period from 02/01/2008 to 28/12/2013 or 1563 observations. These two aspects are chosen because of their different characteristics of moments.

Descriptive analyzes and preliminary tests

The first graph shows the logarithmic changes in the returns of the Total and Bouygues shares.

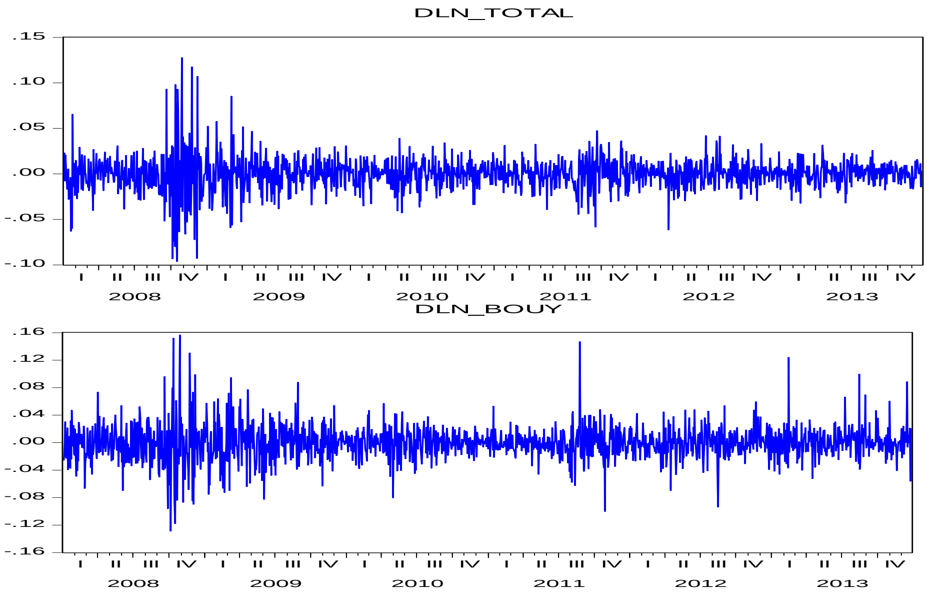


Figure 1 - Evolution of TOTAL and BOUYGUES yields

They exhibit non-stationarity confirmed by unit root tests. To stationarize our series, we retain the first differences in the log of prices that are approximations of the financial returns of our selected securities.

The assets withheld fell sharply following the 2008 crisis. This decline, the largest ever, is the result of a combination of negative performance, unitholder disinvestments, and liquidation of units. The portfolio manager makes purchases and sales on the assumption that changes in asset prices are composed of a market trend and a specific asset factor. It hedges its portfolios by buying undervalued assets and selling overvalued assets.
The descriptive statistics in Table 1 indicate that yields are volatile, leptokuretic and asymmetric: the distributions of returns are not Gaussian distributions. The analyzes of the stationary price series reveal other characteristics of the financial series: no autocorrelation of the returns but autocorrelation of the yields squared, asymmetry and leptokurticity of the distribution of the returns, clusters of volatility. In addition to the highlighting of ARCH effects, GARCH the application of the BDS test rejects the hypothesis of linear structures.

These shortcomings of linear models lead to consider a nonlinear approach to the process generating series of returns. To justify this choice we retain tests that make it possible to test if a series is i.i.d. The test results are given in the appendices for different epsilon values and for different dimensions.

Statistics	Total	Bouygues
Average	0.03527903	-0.000420461
Standard deviation	0.9996982	0.02478897
Min	-1	-0.1287917
Max	1	0.1566574
Skewness	-0.070602	0.4575874
Kurtosis	1.004985	8.315181

Table1- Descriptive Daily Performance Statistic

The moments of order three and four of the two returns are different: the asymmetry is less than o for the title Total, while it is greater than o for the title Bouygues. The Bouygues fruetons are more important than Total's. These differences explain the choice of these titles for this study.

Tests	Total	Bouygues
Autocorrélation	3.11%	3.11%
Autocorrélation au carré	21.12%	21.2%
Jarque–Bera (p-value)	2.2e-16%	2.2e-16%

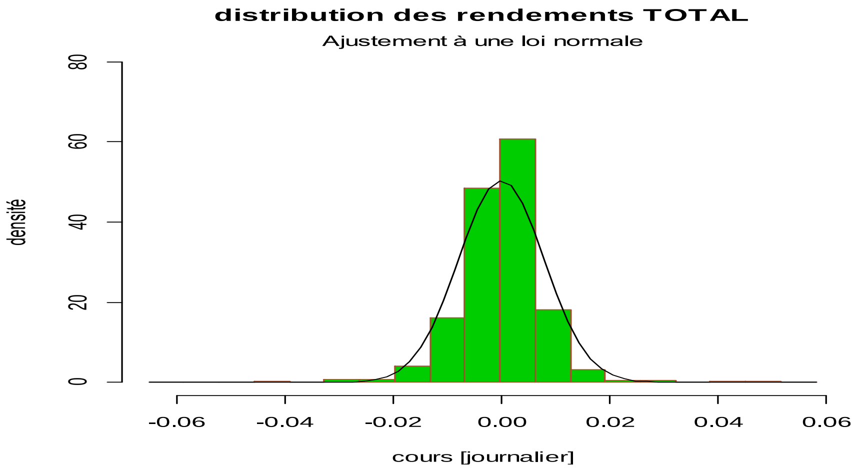
Ljung–Box (p-value)	0.3486 %	0.1559 %
Ljung–Box squared returns (p-value)	0.0%	2.2e-16%

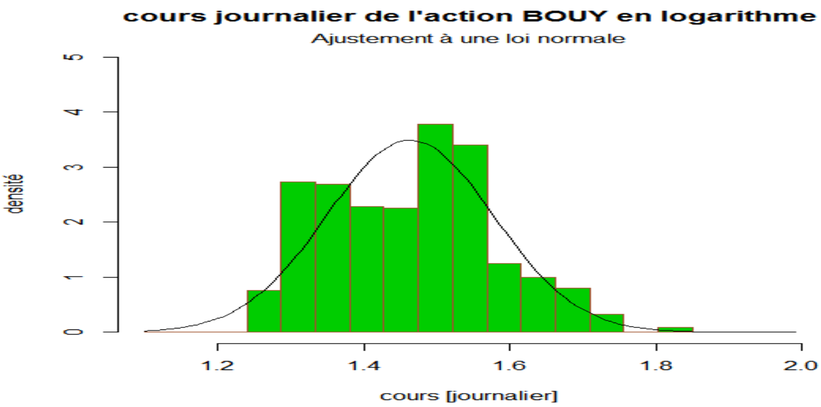
Table 2-Statistical Tests of JB

In general, autocorrelation is used to characterize linear dependencies in residual series (time series corrected for trend and seasonality). Indeed, the trend and the season are deterministic components. Moreover, if the studied series has characteristics that evolve over time, it can be difficult to estimate its statistical properties because one generally has only one realization of the process which is not enough to make the estimate. However, it is very useful to understand how the empirical autocorrelation of a raw series with a trend and / or seasonality will look.

3. Empirical Modeling of VaR

In order to take into account the preceding characteristics of our series of yields, we propose to model their distributions using the approximations presented previously (Gram Charlier, the extension of Cornish Fisher, and Jonson).
The graphs below represent the empirical distribution of returns for each asset, with the adjustment of the normal distribution of the same mean and standard deviation.





These graphs clearly show the problem of leptokurticity at the tails of distributions. To compare our different approximations we use the values of the VaR calculated according to the different approaches.

The quantiles of 1%, 5% and 10% are maximum losses at 99%, 95% and 90% probability respectively. The minus sign means a loss (left part of the distribution). On the other hand, the quantiles of the 90%, 95% and 99% are maximum gains at respective probabilities of 10%, 0.5% and 0.1%. The positive value means a gain (right part of distribution).

	R.Total			R.Bouygues		
	Normale	GC	CF	Normale	GC	CF
VaR(1%)	0,04219	-0,212	-0,06	-0,0366	-0,555	-0,61
VaR(5%)	-0,02976	-0,13252	-0,0276	-0,02585	24,67103	0,099887
VaR(10%)	-0,0231	0,022471	-0,0147	-0,02011	16,49001	0,240384
VaR(90%)	0,023609	0,02295	0,0151	0,020324	18,1094	-0,216
VaR(95%)	0,030236	0,13265	0,0281	0,0260568	25,0964	-0,035
VaR(99%)	0,042665	-0,214	0,0692	0,036809	-1,579	0,7676

Table 3 - Normal VaR, GC and CF at 1%, 5%, 10%, 90%, 95%, and 99%

VaR Jonson	Loi	Total	Bouygues
1%	Normale	-13.009	-879.34
	Log-Normale	-6.718	-85.668
	Borné	-23.9	-119
	Non Borné	3.142	389.792
	Normale	-16.188	-1035.6

5%	Log-Normale	-5.399	-73.724
	Borné	-12.77	-96.5
	Non Borné	5.391	473.889
	Normale	-18.555	-104.88
10%	Log-Normale	-4.711	-72.881
	Borné	-9.49	-95
	Non Borné	6.918	480.892
	Normale	-18.607	-1160
90%	Log-Normale	-4.697	-66.530
	Borné	-9.44	-84
	Non Borné	6.951	539.681
	Normale	-16.228	-1180.1
95%	Log-Normale	-5.3859	-65.512
	Borné	-12.7	-82.32
	Non Borné	5.4175	550.230
	Normale	-13.038	-1220.2
99%	Log-Normale	-6.703	-63.577
	Borné	-23.7	-79.1
	Non Borné	3.163	571.267

Table 4 - Jonson VaR under different families at 1%, 5%, 10%, 90%, 95% and 99%

VaR approaches based on the Cornish-Fisher, Gram Charlier, and Jonson developments aim at modifying the multiple associated with the normal distribution in order to integrate the third and fourth moments of the yield distribution. These approaches make it possible to obtain an approximate analytical expression of the quantile of a distribution as a function of its moments.

By limiting the three approaches mentioned above to its first terms, we obtain an analytic expression of the VaR involving the expectation μ, the standard deviation σ, the Skewness and the Kurtosis of the returns.

The tables below show the degrees of asymmetry, the kurtosis levels and the w_α statistic calculated from the above equations (Gram Charlier, Cornish-Fisher, and Jonson approximations) for the 1%, 5% and 5% thresholds. %, 10%, 90%, 95% and 99%.
As can be seen from the tables, excess kurtosis slightly dominates the asymmetry in the calculation of the Cornish-Fisher expansion. By using the 1% threshold as a multiple in the Gaussian VaR equation, the risk of these assets is greatly underestimated.

We also note that the statistics (quantile w_α) of all the thresholds of the Jonson approach are very weak compared to the two previous ones, explained by the dynamism of this approach.

The estimate of losses with VaR Gram Charlier is generally close to the VaR Cornish Fisher. The Gaussian VaR gives losses greater than those of the Cornish Fisher VaR and the VaR Gram Charlier, this is valid for both titles. Cornish-Fisher VaR results in results far removed from Gaussian VaR and overstates losses in all cases.

The results are presented by quantiles (see appendices), for each model. Each asset can be considered as an example of how VaR calculations compare from one model to another.

Quantiles of 1%, 5% and 10%
Quantiles of 1%, 5%, and 10% are maximum losses at 99%, 95%, and 90% probabilities. The minus sign means a loss (left part of the distribution). We grouped the three models (Gaussian, Gram Charlier and Cornish Fisher) under different loss thresholds in the same table, to make our comparison. The differences between the results of the models at the different thresholds are reduced, as regards the securities whose distributions are more or less symmetrical.

Quantiles of 90%, 95% and 99%
For the same purpose as the previous table, but this time we are talking about gains rather than losses. The quantiles of 90%, 95% and 99% are maximum gains at probabilities of 10%, 5% and 1%. The plus sign means a gain (right side of the distribution).

Again, the VaR Gram Carlier and VaR Cornich Fisher are close.
Gaussian VaR gives gains lower than those obtained with Gram Carlier VaR, and Cornich Fisher VaR in most cases and more particularly with highly asymmetric distributions. The Cornish Fisher VaR gives even greater gains than those estimated with Gaussian VaR. The differences are larger for the 99 percent, 95 percent and 90 percent quantile calculations. Same observation on the left and right tails of the distribution: the gaps are accentuated while moving away from the center.

VaR Jonson: Quantiles of 1%, 5% and 10%
Quantiles of 1%, 5% and 10% are maximum losses at 90%, 95% and 99% probabilities. The minus sign means a loss (left part of the distribution). The VaR Jonson calculated according to the families of the following laws: Normal family, Log-Normal family, Borné family, and non-Borne family.

The Jonson VaR calculated according to the log-normal family and the Cornish Fisher VaR are close. The Jonson VaR with the Borné family results in higher earnings than the Jonson VaR of the Non-Borne family in most cases. In general, the results are closer in the case of the Jonson VaR with the Borné family and the Jonson VaR with the Normal family (distributions whose shape parameters are close to zero).
VaR Jonson: Quantiles of 90%, 95% and 99%

The quantile of 90%, 95% and 99% are maximum gains at probabilities of 10%, 5% and 1%. The plus sign means a gain (right side of the distribution). In the calculation of the VaR according to the Jonson models, the differences are larger for the calculations performed at the different quantile levels. Same observation on the left and right tails of the distribution: the gaps are accentuated while moving away from the center.

VaR with Moments of order three and four at the threshold of 5%

While keeping our respective series as such, we assigned different values to the skewness and kurtosis parameters, in order to see the impact of this change on the calculation of VaR. The calculation of VaR for Total and Bouygues yields is made at the 5% threshold.

In the tables (8.1 and 8.2) below we see the results on the different models, we clearly see that, the further away from the position of normality is to say skewness = 0 and kurtosis = 3, plus the risks become uncontrollable.

Skewness	-1			0			1		
Kurtosis	VaR GC	VaR CF	VaR J.log	VaR GC	VaR CF	VaR J.log	VaR GC	VaR CF	VaR J.log
	-	-		-	-		-		
	0,425	0,025		0,406	0,030		0,389		
	-	-	0,026	-	-	0,419	-		
	0,338	0,025	1,636	0,320	0,030	1,738	0,302		
	-	-	-	-	-	-	-	-0,036	
0	0,251	0,024	25,18	**0,233**	**0,029**	**5,018**	0,216	-0,035	1,003
1,5	-	-	-	-	-	-	-	-0,035	-0,794
3	0,165	0,024	9,498	0,147	0,029	4,485	0,129	-	-0,962
4,5	-	-	-	-	-	-	-	0,0342	-1,326
5,5	0,107	0,023	8,532	0,089	0,029	4,568	0,072	-0,034	-1,575

Table 7.1- Total VaR: for moments 3 and 4 fixed

Skewnes s	-1			0			1		
Kurtosis	VaR GC	VaR CF	VaR J.log	VaR GC	VaR CF	VaR J.log	VaR GC	VaR CF	VaR J.log
	-	-		-	-		-		
	0,718	0,02		0,68	0,02		0,658	-	
	7	2	-	8	6	0,0007	3	0,03	1,00088
	-	-	0,9993	-	-	4	-	1	7
	0,572	0,02	1,2514	0,54	0,02	12,647	0,511	-	-
	2	2	2	2	6	6	8	0,03	0,56546
	-	-	-	-	-	-	-	1	-
0	0,425	0,02	23,229	**0,39**	**0,02**	**4,3949**	0,365	-0,03	0,70513
1,5	8	1	-	**5**	**5**	-	4	-0,03	-
3	-	-	8,4548	-	-	3,9087	-	-	1,01969
4,5	0,279	0,02	-	0,24	0,02	-	0,218	0,02	-
5,5	3	1	7,5822	9	5	3,9762	9	9	1,23575

	-	-0,02	-	-	-
	0,181		0,15	0,02	0,121
	6		1	5	3

Table 7.2- Bouygues VaR: for moments 3 and 4 fixed

The color values are the results of the different models in accordance with the criteria of normality.

Gaussian VaR is the one that gives the lowest loss estimates, especially for the most extreme quantiles, except where the distributions are symmetrical (shape parameter close to zero).

The development of Cornish Fisher is not satisfactory. Cornish Fisher VaR systematically gives higher estimates of losses and gains than Gram Charlier VaR and VaR calculated using the Jonson models. Jonson VaR models are generally close to each other. In a word we can say the approach developed from extension of Jonson is close to reality.

Backtesting

The classic approach used by many authors is to provide VaR forecasts taking into account the long position, ie for negative returns. However, the forecast capacity of the models that are proposed must be evaluated in long position but also in short position. Actors participating in the financial markets are not only curious to know the maximum loss that may be caused by a fall in the price of the assets they hold, but they can, in a short position, worry about the maximum increase in the price of an asset that they intend to acquire. We present the results of backtesting tests in short and long positions, for conditional and unconditional coverage, both in the sample and out of the sample.

Rather than compare the calculations of a model to the realizations, we make the decision to proceed by simulation by generating a quantity of scenarios as important as desired. We implicitly assume that the Jonson approach computed according to the log-normal law family is the true model because it is the one that uses the most information about the distribution tails. We then measure the errors that other methods of calculating VaR produce. If these calculation methods generate significant errors, this will validate the relative contribution of the moments of order 3 and 4.

In this backtest, we will test the validity of the VaR levels computed above by simulating data in the law of each title estimated through the three- and four-order moments and calculating the number of times, on average, where the VaR are exceeded. The standard deviation of these overshoot numbers is also calculated.

We produce N = 1000 data for each asset and calculate the average values of the exceedances and the corresponding standard deviations.

These analyzes are performed for quantiles of 1%, 5%, 10%, 90%, 95% and 99% respectively.

Backtesting Application

The tables below show, by quantile, the mean and standard deviation of the number of VaR exceedances specified for each model.

Surcharge for quantile of 5%

Action	VaR Gaussienne		VaR G. Charlier		VaR C. Fisher		VaR Jonson	
	Average	S.Dev	Average	S.Dev	Average	S.Dev	Average	S.Dev
Total	0,505	0,49997	0,562	0,4961	0,505	0,4999	0	0
B&Y	0,505	0,49997	1	0	0,552	0,4972	0	0

Table-8.1: Comparative table of VaR at the 5% threshold

Gaussian VaR underestimates or overestimates losses depending on the case. Losses are generally overestimated in the case of the Bouygues share, whose distribution is highly skewed.

The development of Cornish Fisher generally overestimates losses (overruns often greater than 0.5).

NB: By construction of the simulation, the number of overruns for the Jonson VaR is zero. This is only a numerical consequence of the simulation, so we notice that this model has not recorded any overruns.

Surcharge for quantile of 95%

Action	VaR Normale		VaR G. Charlier		VaR C. Fisher		VaR Jonson	
	Average	S.Dev	Average	S.Dev	Average	S.Dev	Average	S.Dev
Total	0,528	0,526	0,562	0,49614	0,527	0,49927	0	0
B&Y	0,49921	0,499	1	0	0,5	0,5	0	0

Table-8.2: Comparative table of VaR at the 95% threshold

Gaussian VaR most often overestimates earnings (overruns often less than 0.5). The results are more in line with the true model for the most symmetric distribution (Total case in particular).

The Cornish Fisher VaR gives unsatisfactory results with overruns sometimes significantly higher or lower than the expected level meaning alternatively underestimation or overestimation of earnings except for Total at a more or less symmetrical distribution.

The overflow tables for quantile residues are listed in the Appendices.

Gaussian VaR model error is important, leading in most cases to underestimate losses and overestimate gains. The backtest also leads to the rejection of VaR estimates made from the Cornish Fisher development as this model tends to overestimate losses.

4. Conclusion

This research work has focused on many elements that come into play when designing and evaluating a risk measure in finance. The crisis reminded us of a simple teaching but that the habit of prosperity had managed to conceal: quick and safe enrichment does not exist. Growth in the value of goods is limited by time and risk.

Value-at-Risk (VaR) is commonly used by regulators and practitioners to manage exposures to market risks. In the different sections, we studied the performance of different methodologies used to measure VaR. In this way we have found that among our different methodologies, the approach using the normal law is the least accurate and without any surprise. Applied research has developed formulas to compensate for the inability of Gaussian VaR to adapt to the asymmetric distribution of returns on financial assets. Notably, the Gram Charlier method, the development of Cornish Fisher and the extension of Jonson are corrections of the Gaussian VaR formula of introducing kurtosis and skewness into its expression.

Considering it necessary to identify a model that adapts correctly to the shape of the tails of distributions, we have exploited the properties of tools developed by modern statistics, the moments of order three and four. Their properties make it possible to better capture information on extreme values. The estimation of the models makes it possible to build a more robust indicator of the VaR. It is a Parametric Jonson VaR set on the log-normal family that has four parameters.

The use of Johnson's methodology with the third and fourth order moments manages to better measure the risk than the normal law in general. These results are consistent with the literature and demonstrate the relevance of using it rather than the normal law.
In addition, several backtests show that: Jonson VaR is a more accurate model than Gaussian VaR and gives results that are both fairer and more stable than the Cornish Fisher VaR and the Grlier Charlier VaR. The VaR Jonson is therefore a great progress. The calculation of the VaR is then very theoretical and the real losses are higher than expected. Taking into account the disappearance of the market in the VaR models would thus constitute a significant progress.

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