TIME PREFERENCES AND THE PROPERTY RIGHTS PARADIGM

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Abstract

Capitalism relies heavily on property rights to resolve conflicts over the use of scarce resources. Property rights are defined in the literature as the expected ability of an economic agent to use an asset (Allen 1999; Barzel 1997; Lueck and Miceli, 2005; Shavell, 2002). A systematization of the economic analysis of property rights is due to Demsetz (1967) and Alchian and Demsetz (1973), whose 'property rights paradigm' has become, among contemporary economists, the 'classical view' on property rights and economic incentives. According to Alchian and Demsetz private property rights represent always a social institution that creates incentives to efficiently use assets, and to maintain and invest in assets. In particular private property rights allows for the internalization of the externality existing in the communal right system, where any owner cannot exclude the others from enjoying the fruits of her effort and hence no one has any incentive to use inputs that have a future payoff. This view has a strong appeal among contemporary economists (see for example Glaeser and Schleifer, 2002; Djankov et al, 2003). As a consequence, the role of the State in codifying and enforcing the property rights on productive assets is generally considered as crucial to promote investment and growth, even if it may entail some public costs. In this work I question the conclusion that (private) property rights security, defined as the expected ability of an economic agent to use an asset, has always a positive effect on investment incentives. Time preferences matter. I develop an analytical framework to analyse the interactions of property rights and investment incentives grounding on the model of quasi-hyperbolical discounting originally proposed by Phelps and Pollak (1968) in the context of intergenerational altruism and then used by Laibson (1994, 1997) to model time-inconsistency within an individual. Moreover, I adopt a general solution concept called 'perception-perfect strategy' proposed by O'Donoghue and Rabin (1999, 2000). In this setting I show that while the expected ability to enjoy the benefits from the investment (what we call property rights security on investment) always affects positively investment incentives, the expected ability to use an asset (what we call property rights security on asset) has a non-negative effect on investment incentives only under the hypothesis that investors are time-consistent exponentially discounters. Instead we show that, under the more general and empirically supported hypothesis of hyperbolic discounting, property rights security on investment but property rights insecurity on asset maximize investment incentives.

1. Introduction

Property rights consist in the expected ability of an economic agent to use an asset (Barzel, 1997; Lueck and Miceli, 2005). A seminal contribution about the economic effects of property rights is due to Blackstone. In his "Commentaries on the Laws of England" (1765-1769), the author emphasizes the economic virtues of private property rights on land as it turned out in England: "*And the art of agriculture by a regular connexion and consequence, introduced and established the idea of a more permanent property in the soil, than had hitherto been received and adopted. It was clear that the earth would not produce her fruits in sufficient quantities, without the assistance of tillage: but who would be at the paints of tilling it if another might watch an opportunity to seize upon and enjoy the product of his industry, art and labour*? (Blackstone, 1765-1768, Book II, Ch. 1). The systematization of the above intuition is due to Demsetz (1967) and Alchian and Demsetz (1973), whose approach became, among contemporary economists, the "classical view" about the economic incentives of property rights. It can be summarized as follows. Consider the simple case of a utility-maximizing individual who has to decide whether or not to undertake a certain investment on an asset, leading him to incur in an immediate cost and enjoy some expected future benefits. Suppose that property rights are not fully guaranteed, meaning that there exists a positive probability that in the future some other agent embezzles the fruits of the investment. This risk of expropriation represents a random tax on the benefits of the investment which, *ceteris paribus*, lowers the expected value of the investment and hence the incentives to invest. Alchian and Demsetz derive from the previous reasoning the following conclusion: private property rigths security *always* represent an institution that creates incentives to efficiently use assets, and to maintain and invest in assets. This view is largely shared among contemporary economists. As pointed out by Kaplow and Shavell (2002), "*Today the virtues of property rigths seem to be taken for granted or are only casually asserted*". In this paper we discuss the generality of this conclusion focusing our attention on how some subjective characteristics of individuals, in particular their time preferences, can have an impact on the role exercised by the institution of property rights.

Time preferences represent crucial determinants of investment behaviour. As discussed by Irving Fisher (1930), impatience is more than a rare behavioural trait of investors: "*Generally speaking, the greater the foresight, the less the impatience and vice versa…This is illustrated by the story of the farmer who would never mind his leaking roof. When it rained he could not stop the leak, and when it did not rain there was no leak to be stopped! Among such person, the preference for immediate gratification is powerful because their anticipation of the future is weak*" (Fisher, 1930, p.81). Recent empirical and experimental evidence (for a survey see Frederick, Lowenstein and O'Donoghue 2002) has shown how impatience (present-biased preferences) is a widespread and standard behavioural characteristic and not an exception. For this reason the more recent literature on time preferences (Phelps and Pollak, 1968; Laibson, 1997) maintains that Samuelson's model (1937) of time preferences, based on exponential discounting, must be seen as a particular case of a more general form (quasi-hyperbolic discounting), where individuals may be present-biased. Notice that impatience, differently from exponential discounting, implies the propensity to change preference orderings at different points in time, i.e. time-inconsistency. A contemporary

example of such a behaviour can be seen in Brazilian landlords facing movements such as Movement of Sem Terra which occupies unused lands: why do landlords not make them productive or do not sell them?

The main point of this paper is that if we assume the possibility of present-biased preferences of investors the role of property rights must be re-discussed in a broader way. In order to analyse in a rigorous way this feature, we construct a framework where we can discuss the impact of property rights on investment incentives under different time preferences. In this model we use two different variables to capture property rights on used assets, i.e. as asset on which an investment has made, and property rights on unused assets, i.e. an asset on which an investment has not been made (yet). The conclusions we draw from our model are twofold. First, property rights security on used assets always positively affects investment incentives for any investor regardless her time preferences; second, property rights security on an unused asset, i.e. the ability to make the investment on a certain asset in future periods, has no impact on the behaviour of a time-consistent investor but plays a role of disincentive for present-biased investors: indeed since these investors may delay or even procrastinate (i.e. continuously delay and never make) a profitable investment simply because of impatience, a higher security in the future control of the asset can discourage the investment itself. In this case, then, the virtue of property rights cannot be taken as granted.

The paper is organized as follows. In order to discuss the relationship between time preferences, property rights and investment, in paragraph 2 we build a model of land investment and formalize the solution "perception-perfect strategy" (0'Donoghue and Rabin, 1999; 2000). In paragraph 3 we explore the relationship between property rights security and investment incentives under alternative time preferences. Finally, in section 4, we draw some concluding remarks.

2. The Model

2.1 The Basic Setting

Consider an infinitely living landowner who faces the decision of whether or not to undertake a certain feasible investment[1] on a field (asset) of size $0 < \bar{L} < \infty$. In each time $t = 1,2, ...$ the landowner makes a schedule specifying the share of the field she will invest on in each period from t onwards. Let $0 \leq L_t \leq \bar{L}$ the share of the field on which the investment is made at time t. I assume that, for a given level of investment L_t, the individual bears a cost $C(L_t)$ in time t and reaps an infinite stream of benefits $B(L_t)$ from period $t + 1$ onwards. The landowner is credit-constrained[2]: she bears the cost of the possible investment by reducing her consumption and/or leisure.

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[1] In this setting there is one and only one investment which may be undertaken. We can think of any kind of investment on land such as planting, fertilizing, introducing a technological innovation, but also selling a piece of land in presence of transaction costs. In this paper I do not enter the issue of how time preferences may affect the choice of optimal investment when a menu of alternatives is faced.

[2] The hypothesis that the landowner is credit-constrained is crucial in our analysis. Indeed, the possibility to have access to the credit may be a way to overpass not only the liquidity problem, but also self-control problem due to present-biased preferences. We leave to further research the aim to study the interaction between time preferences, credit and investment incentives. However we could say that the majority of small and medium in low-income countries landowners are credit-constrained.

Property rights on land may be partially insecure. For the sake of my purpose, I consider and model separately the expected ability to enjoy the benefits from investment on land (what I call *property rights on investment*) and the expected ability to invest on the still unused asset (*property rights on unused asset*).[3] I assume that, in each time $t \geq 1$, the investor faces a probability $0 < p \leq 1$ that in each future period she will keep enjoying the benefits stemming from past investment and a probability $0 < q \leq 1$ she will keep the possibility to undertake the investment on the still unused land. Correspondingly, in each time $t \geq 1$ the investor faces a probability $0 \leq 1 - p < 1$ that in each future period she will be expropriated of the benefits from past investment and a probability $0 \leq 1 - q < 1$ to be expropriated from the unused land. For the sake of simplicity and without loss of generality, I make the simplifying assumption that the investor is risk-neutral and that if the rights to enjoy the benefits from past investment or to invest on the unused land are interrupted, they are interrupted forever.

I model the investor's intertemporal preferences in terms of "quasi-hyperbolic discounting" (Phelps and Pollak, 1968; Laibson, 1997), to capture the possibility of present-biased and time-inconsistent preferences. Let u_t be the instantaneous utility at time t and Eu_τ the expected utility at the generic future period τ. The inter-temporal utility perceived at time t by a risk-neutral individual is represented by the following utility function: $EU^t(u_t, u_{t+1}, \dots) = u_t + \beta \sum_{\tau=t+1}^{\infty} \delta^{\tau-t} Eu_\tau$, where the parameter $0 < \delta < 1$ is a long-run discount factor and $0 < \beta \leq 1$ represent time-inconsistent preferences for immediate gratification. Notice that, for $\beta = 1$, the standard form of exponential and time-consistent preferences is obtained. If, instead, $\beta < 1$, the investor is characterized by short-term impatience, i.e. she has an extra bias for the present over the future.

In this setting, assuming that costs and benefits enter linearly the utility function, a landowner who is characterised by a long-run discount factor $0 < \delta < 1$, deals with impatience attitudes $0 < \beta \leq 1$, faces a degree of property rights security on investment p and who invests L_t at time t will enjoy an expected utility from this investment equal to[4]: $-C(L_t) + \beta \frac{\delta p}{1 - \delta p} B(L_t)$[5]. In order to analyse the optimal investment behaviour over time, I need to make some assumptions on how the investor perceives her own future preferences. Let $\hat{\beta}$ the investors's belief about her future impatience attitudes β. A present-biased investor (characterized by $\beta < 1$) is called "naïve" if she believes that in the future she will be time-consistent $\hat{\beta} = 1$, whereas she is "sophisticate" if she exactly predict her future preference for immediate gratification ($\hat{\beta} = \beta$). The intermediate case is represented by a partially naïve investor (O'Donoghue and Rabin, 2001), who knows that in the future she will have a preference for immediate gratification but she underestimates their magnitudes ($\beta < \hat{\beta} < 1$).

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[3] In this simple model property rigths security on investment coincides with property rights security on the used asset, i.e. on the part of the asset on which the investment has been made. Notice that no further investment can be made.

[4] Notice that property rights security on investment enters the utility function simply by lowering the long-run discount factor. Instead property rights security on unused asset does not affect the expected utility perceived from the investment (the profitability of the investment) but only the optimal timing in making the investment.

[5] We are assuming that costs and benefits enter linearly the utility function.

2.2 Solution Concept

A compact solution form for the intra-personal game spelled out in the previous paragraph is provided by O'Donoghue and Rabin (1999, 2001) and it is called *perception-perfect strategy*. This solution requires that, at each time t, the investor chooses the level of investment L_t which maximizes her current preferences given *dynamically consistent beliefs*. Within my specific setting, it can be formalized as follows. Let $0 \leq h_t = \sum_{\tau=1}^{t-1} L_t \leq \bar{L}$ be the history at time t. In words, in a given period t the investor faces a history represented by the part of the field on which the investment has been made in previous periods. Moreover, let $A_t \equiv [0, \bar{L} - h_t]$ be the set of actions available in each period t following history h_t. An action $L_t \in A_t$ corresponds to undertaking in time t the investment on a fraction of land of size L_t. I describe individual behaviour by a strategy $s = (L_1, L_{2,}, \dots)$ which specifies for each time t, given history h_t, an action $s(h_t, t) \in A_t$. Moreover let s^0 be the strategy $s = (0,0, \dots)$ and set $\tau(s) \equiv min\{t/L_t \neq 0\}$, where $\tau(s) = \infty$ if $L_t = 0$ for all t. Let us now define as V^t the individual preferences in period-t over current actions and given history h_t, conditional on following the strategy $s = (L_{t+1}, L_{t+2}, \dots)$ from time $t+1$ onwards:

$$V^t = B(h_t) - C(L_t) + \beta \sum_{\tau=1}^{\infty} \left[\delta^\tau p^\tau B(h_t) + \delta^\tau \left(\sum_{i=1}^{\tau} q^{i-1} p^{\tau+1-i} B(L_{t+i-1}) \right) - \delta^\tau q^\tau C(L_{t+\tau}) \right]$$

The expected utility of an investor in time t depends on her instantaneous utility and on the future discounted expected utility from period $t+1$ onwards. The instantaneous utility is given by the benefits reaped in time t from past investment, $B(h_t)$, minus the cost of the investment made in time t, $C(L_t)$. The future utilities depend on the level of past, present and future investment, on the cost and benefit functions, respectively $C(\cdot)$ and $B(\cdot)$, on the short-run and long-run discount factors β and δ and on the degree of property rights security on investment (p) and on unused asset (q).

Let us now define within this framework the concept of dynamically consistent beliefs and perception-perfect strategy.

Definition 1. A strategy $\hat{s}$ represents $\hat{\beta}$-dynamically consistent beliefs if

$$\hat{s}(t, h_t) = arg \max_{L_t \in A_t} V^t \big(C(\cdot), B(\cdot), \bar{L}, h_t, \hat{\beta}, \delta, p, q \big) \text{ for all } t \geq 2 \text{ and } h_t.$$

Dynamically consistent beliefs are characterized by two forms of consistency. First, *internal consistency*: for all contingencies, strategy $\hat{s}$ specifies a sequence of investment which is optimal given the beliefs for the levels of investment in the future periods. Secondo, *external consistency*: at all times $t < \tau$, the investor has the same belief about her behaviour in period τ following history h_τ. This means that the beliefs of an investor can be simply represented by the vector of her frst period beliefs on the levels of investment in all subsequent periods: $\hat{s} = (\widehat{L_2}, \widehat{L_3}, \dots)$.

I can now provide a formal definition of perception-perfect strategy. In addition to the equilibrium condition stated by O'Donoghue and Rabin (2001), I also require the feasibility of the strategy given the constraints represented by the asset endowment $\bar{L}$.

This constraint to land investment is captured by the set of actions available in each period t.

Definition 2. A strategy s^{pp} is a perception-perfect strategy if there exist $\hat{\beta}$-consistent beliefs $\hat{s}$ such that $s^{pp}(h_t, t) = arg \max_{L_t \in A_t} V^t\left(C(\cdot), B(\cdot), \bar{L}, h_t, \hat{s}, \beta, \delta, p, q\right)$ for all $t \geq 2$ and h_t.

A perception-perfect strategy $s^{pp} = \left(L_1^{pp}, L_2^{pp}, \dots\right)$ represents the actual sequence of investment of an investor who maximizes her current preferences given dynamically consistent beliefs.

Finally, I provide here an additional definition which is useful in the following steps.

Definition 3. A level of investment $0 < L_t \leq \bar{L} - h_t$ is β −worthwhile in t if and only if $-C(L_t) + \beta \frac{\delta p}{1 - \delta p} B(L_t) \geq 0$.

In other terms a level of investment L_t is β −worthwhile if the investor perceives in t a non- negative expected utility from this investment.

2.3 Equilibria

In this setting two relevant questions exist about the behaviour of a certain investor; first: when, if ever, does she undertake a positive investment? Second: which amount of land does she invest on? Since in this paper I aim at focusing on the first problem, I make here a simplifying assumption which allows to obtain a simple solution of the second problem without compromising the qualitative analysis of the first one.

Assumption 1. $C(L_t) = cL_t$ and $B(L_t) = bL_t$, where $c > 0$ and $b \geq 0$.

Assumption 1 implies that in this setting there are constant returns to scale from the investment. Given this hypothesis, a feasible positive level of investment $0 < L_t \leq \bar{L} - h_t$ is β −worthwhile in t if and only if $-c + \beta \frac{\delta p}{1 - \delta p} b \geq 0$. Therefore either *any* positive level of investment is β −worthwhile or no positive level of investment is β −worthwhile.

Lemma 1 and 2 will prove that, given this assumption, at any time either the investment is taken on the overall asset $\bar{L}$ or no investment is undertaken. Therefore this framework allows to abstract from any problem concerning the optimal smoothing of a profitable investment across different periods. Let us now provide some insights about the first problem, i.e. the timing of the investment. Intuitively, as I will prove in Lemma 2, if no positive level of investment is β −worthwhile, a person prefers to never undertake the investment simply because her expected utility from the investment is not positive given her time preferences and the degree of property rights on investment she faces[6]. Moreover, in this setting, an investor may never undertake a positive investment even if it is β −worthwhile because of *procrastination*, i.e. a continuous delay.

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[6] Notice that the degree of property rights security on unused asset (captured by the parameter q) does not affect the profitability of the investment; instead, as I will show further on, it affects the decision of "when" undertaking a profitable investment.

Consider for a moment a simplifying case in which the only feasible level of investment corresponds exactly to $\bar{L}$: at each time a landowner either invests on $\bar{L}$ or does not invest at all[7]. Suppose that $\bar{L}$ is β −worthwhile for a certain investor given her time preferences and the degree of property rights security on investment she faces. This investor has a maximum tolerable delay d^* such that, for any $d > d^*$, she prefers making the investment $\bar{L}$ today rather than in d periods. In particular, an investor characterized by self-control problems β, long-run discount rate δ, degree of property rights security on investment p and degree of property rights security on unused asset q will tolerate a maximum delay given by:

$$d^*(\beta) \equiv max\left\{d \in \{0,1,\dots\}: -c\bar{L} + \beta\frac{\delta p}{1 - \delta p}b\bar{L} \leq \beta(\delta q)^d\left[-c\bar{L} + \frac{\delta p}{1 - \delta p}b\bar{L}\right]\right\}$$

Notice that, since production opportunities are characterized by constant returns to scale, $d(\beta)$ does not depend on the level of investment $\bar{L}$ but only on the parameters identifying time preferences (β and δ), production opportunities (c and b) and property rights security (p and q).

In case $d^*(\beta) = 0$, the investor cannot tolerate any delay and she undertakes the β −worthwhile investment $\bar{L}$ immediately. If instead $d^*(\beta) > 0$, the investor might delay the β −worthwhile investment, depending on her beliefs about when in the future she would undertake the investment $\bar{L}$. Consider the case of an investor with self-control problems $\beta < 1$ and fully sophisticated ($\hat{\beta} = \beta$), meaning that the investor perfectly predicts her future behaviour. In this case, at each period t, when the investor plans to undertake the investment, it must be that delaying would imply a total delay of $d^*(\hat{\beta}) + 1 = d^*(\beta) + 1$ periods: hence she will plan to undertake the investment $\bar{L}$ exactly every $d^*(\beta) + 1 = d^*(\hat{\beta}) + 1$ periods and there are $d + 1$ perceptions perfect strategies (i.e. multiple equilibria). If, for example, $d^* = 2$, there are three perception-perfect strategies: $(\bar{L}, 0, 0, \dots)$, $(0, \bar{L}, 0, \dots)$ and $(0, 0, \bar{L}, \dots)$.

Consider now the case of a partially naïve investor characterized by self-control problems $\beta < 1$ and by perceptions of future self-control problems $\hat{\beta}$, where $\beta < \widehat{\beta} < 1$. Given the definition of $d^*(\beta)$, it must be that $d^*(\hat{\beta}) \leq d^*(\beta)$. A partially naïve investor believes that, if she delays now she will tolerate a delay of at most $d^*(\hat{\beta}) + 1$ periods. Therefore, if $d^*(\hat{\beta}) + 1 \leq d^*(\beta)$, the partially naïve investor perceives that, if she delays now, she will undertake the investment $\bar{L}$ within a tolerable number of periods and she delays. However, since the same reasoning is iterated at each period, the investor will infinitely delay and never undertake the profitable investment $\bar{L}$, i.e. she procrastinates it. Finally, a fully naïve investor (with $\beta < 1$ and $\hat{\beta} = 1$) believes that if she does not undertake a β −worthwhile investment $\bar{L}$ now, she will undertake it the next period., Therefore she procrastinates the profitable investment $\bar{L}$ as long as $d^*(\beta) > 0$.

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[7] Lemma 2 proves that, given our assumptions, this result holds without any other restriction.

We now provide a formal characterization of dynamically consistent beliefs (Lemma 1) and perception-perfect strategies (Lemma 2)[8].

Lemma 1. Any dynamically consistent belief $\hat{s}$ must satisfy: for each $t \geq 2$, either $\widehat{L}_t = 0$ or $\widehat{L}_t = \bar{L}$. If $\bar{L}$ is not $\hat{\beta}$-worthwhile, $\widehat{L}_t = 0$ for all $t \geq 2$. If $\bar{L}$ is $\hat{\beta}$-worthwhile and $d(\hat{\beta}) = 0$, then $(\widehat{L_2}, \widehat{L_3}, ...) = (\bar{L}, 0, 0, ...)$. If instead $\bar{L}$ is $\hat{\beta}$-worthwhile and $d(\hat{\beta}) > 0$, there are multiple dynamically consistent beliefs.

Lemma 1 states that the only fraction of land one can expect to possibly invest on in the future is the overall land $\bar{L}$, provided that this level of investment is $\hat{\beta}$-worthwhile. Whenever $\bar{L}$ is $\hat{\beta}$-worthwhile and $d(\hat{\beta}) > 0$, the first date of completion is indeterminate and any dynamically consistent beliefs must be cyclical. Hence there are multiple dynamically consistent beliefs: the investor will expect to undertake the $\hat{\beta}$-worthwhile investment exactly every $d(\hat{\beta}) + 1$ periods. If instead $\bar{L}$ is $\hat{\beta}$-worthwhile and $d(\hat{\beta}) = 0$, the first date of completion is not indeterminate: the investor expects to undertake the investment exactly in the second period.

Lemma 2 characterizes formally the set of perception-perfect strategies.

Lemma 2. Any perception perfect strategy s^{pp} must satisfy: for each $t \geq 1$, either $L_t^{pp} = 0$ or $L_t^{pp} = \bar{L}$. If $\bar{L}$ is not $\hat{\beta}$-worthwhile or $\bar{L}$ is $\hat{\beta}$-worthwhile but $d(\hat{\beta}) + 1 \leq d(\beta)$, then $S^{pp} = \{s^0\}$. If $\bar{L}$ is $\hat{\beta}$-worthwhile and $d(\beta) = d(\hat{\beta}) > 0$, there are multiple perception perfect strategies. If instead $\bar{L}$ is $\hat{\beta}$-worthwhile and $d(\beta) = d(\hat{\beta}) = 0$, the unique perception-perfect strategy is $s^{pp} = (\bar{L}, 0, 0, ...)$.

Lemma 2 provides a complete answer to the following question: if an investor decides to undertake a positive investment, which amount of land will she invest on? The answer is clear-cut: given assumptions 1 and 2, if an investor decides to undertake a positive investment, she will invest on all the available land $\bar{L}$ (this is true regardless time preferences). Moreover Lemma 2 provides some answers to the question: when, if ever, does the investor undertake the investment $\bar{L}$? Whether or not she will undertake the investment $\bar{L}$ is fully determined: if $\bar{L}$ is not β-worthwhile, the investor does not undertake the investment simply because it is not profitable given her time preferences. If instead $\bar{L}$ is β-worthwhile and $d(\beta) + 1 \leq d(\hat{\beta})$, the investor does not undertake the profitable investment because she *procrastinates*: she prefers to delay the investment $\bar{L}$ thinking that in the future she will undertake it, but in fact she will not. If instead $d(\beta) = d(\hat{\beta})$, the person must in some period perceive an intolerable delay and will undertake the investment $\bar{L}$. However when exactly she undertake $\bar{L}$ is indeterminate because it depends on her dynamically consistent beliefs, which are multiple. Therefore the model fully determines *which level of investment* the landowner will possibly undertake, *whether or not* she will undertake it and, for the case $d(\beta) = d(\hat{\beta})$, it provides a range of periods within which she will make the investment $\bar{L}$. Notice that, given Lemma 2, we can study the perception-perfect strategies of my model by simply considering that the investor has to decide in each period whether or not making a one-shot investment $\bar{L}$.

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[8] All proofs are in Appendix.

3. Property Rights and Investment Behaviour

In this section I study how property rights affect investment behaviour.

3.1 Property Rights and Time-Consistent Behaviour

Consider the specific case $\beta = \hat{\beta} = 1$: the investor has no bias towards the present and she is perfectly time-consistent. This is the traditional assumption made in the literature on property rights and investment incentives. In this case, the optimal planning made of the investment made at the first time coincides with the optimal planning made in each subsequent period and determines the actual strategy followed by the investor. The following proposition holds:

Proposition 1. For time-consistent investors:

(1.1) $s^{pp} = (\bar{L}, 0, 0, \dots)$ *is the unique perception perfect strategy if and only if $\bar{L}$ is $\beta - worthwhile$,*

otherwise $S^{pp} = \{s_0\}$.

(1.2)   *Property rights security on investment (p) positively affects investment incentives.*

(1.3)   *Time-consistent investors are neutral to property rights security on unused asset.*

Proposition 1 points out that the relationship between property rights and investment incentives under the hypothesis of time-consistent preferences. A time-consistent investor immediately undertakes the investment $\bar{L}$ as long as it is profitable given her long-run discount factor, the production opportunities described by the parameters c and b and the degree of property rights security on investment she faces. If instead the investment is not profitable, a time-consistent investor plans to never undertake it and actually never undertakes this investment. In other terms a time-consistent investor never delays a profitable investment: indeed, since she does not have any bias towards the present utility, delaying a profitable investment would simply mean to enjoy lower expected benefits (in terms of utility now) because of long-run discounting. Proposition 1 also points out the effects of property rights security on investment on the behaviour of time-consistent investor: a lower property rights security would mean to reduce the expected benefits of the investment and hence its profitability. Instead, property rights security on unused asset does not affect the incentives faced by a time-consistent investor. Intuitively the reason is the following: the degree of property rights insecurity on unused asset captures the risk of expropriation of a certain asset on which the investment has not been made. For time-consistent investor, the only reason explaining the fact that an investment has not been made is that this investment is not profitable; but if the investment is not profitable, a time-consistent investor plans to never undertake it and actually never undertakes it. Therefore for a time-consistent investor property rights security on unused asset is not binding. Since the standard view on property rights and economic incentives assumes exponential discounting and time-consistency, the only kind of property rights that matters are those on investment and the only effect of their security on investment incentives is positive. However, as we discuss in the next paragraph, when we consider a more general form of time preferences the situation

changes and the distinction between property rights on investment and property rights on unused asset becomes necessary.

3.2 Property Rights and Present-biased Behaviour

Let us now explore the behaviour of present-biased and time-inconsistent landowners. The following proposition holds:

Proposition 2. For present-biased investors:

(2.1) A β-worthwhile investment $\bar{L}$ may be delayed or procrastinated.

(2.2) Property rights security on investment (p) positively affects investment incentives.

(2.3) Property rights security on unused asset (q) has a negative effect on investment incentives. In particular it increases the delay in undertaking a β-worthwhile and boots the propensity to procrastinate of naive and partially naïve investors.

Proposition 2 highlights the effect of impatience on investment behaviour. While time-consistent investors never delay a profitable investment, present-biased investors may delay a finite number of periods and even procrastinate a β-worthwhile investment $\bar{L}$. Obviously, when $\bar{L}$ is not β-worthwhile no investment is made simply because it is not profitable for the investor given her preference for immediate gratification and the degree of property rights security on investment she faces. If instead the investment is β-worthwhile, a present-biased investor plans to undertake the investment $\bar{L}$ because she deems it profitable. However her preference for immediate gratification tends to lead her to delay the investment. When the present-biased investor will undertake the investment depends on her actual maximum tolerable delay, $d(\beta)$ and the maximum tolerable delay she believes to have in the future, $d(\hat{\beta})$. If $d(\beta) = 0$ the investor cannot tolerate any delay and she will undertake the investment $\bar{L}$ in the first period. If instead $d(\beta) > 0$, two cases are possible. If $d(\hat{\beta}) = d(\beta)$, the investor will undertake the investment within $d(\beta)$ periods, but when exactly this occurs is indeterminate because there are multiple perception-perfect strategies. If instead $d(\hat{\beta}) + 1 < d(\beta)$, the investor procrastinates and thus never undertakes the β-worthwhile investment $\bar{L}$ because of a systematic delay due to underestimation of future self-control problems.

Part (2.2) points out the effect of property rights security on investment, represented by the parameter p, on investment behaviour of a present-biased investor. This effect is positive since if affects the profitability of the investment.

Part (2.3) of Proposition 2 points out how property rights security on unused asset shapes investment incentives of present-biased investors. Property rights security on unused asset, represented by the parameter q, does not affect the profitability of the investment. Instead, it affects the propensity to delay an investment as long as it is profitable. Suppose that the investment $\bar{L}$ is β-worthwhile. As shown in Appendix, when a present-biased investor is sophisticated (i.e. $\beta = \hat{\beta} = 1$), for given production opportunities and time preferences there always exists a unique value $\tilde{q}$ of property rights security on unused asset such that if (and only if) $q \leq \tilde{q}$, $d(\beta) = d(\hat{\beta}) = 0$ and the unique perception-perfect strategy consists in undertaking the profitable investment in the first period. Instead, for $q > \tilde{q}$, there are multiple perception-perfect strategies: what is

determined in the model is that the investment is undertaken within $d(\beta) = d(\hat{\beta})$ periods, where $d(\beta) = d(\hat{\beta})$ is non decreasing in q: the lower is q, the shorter is the maximum delay for a present biased and sophisticated investor to undertake the profitable investment $\bar{L}$.

Let us now discuss the case of partially naïve and fully naïve investors. Also in such a case, a unique value $\tilde{q}$ always exists such that, given the other parameters, $d(\beta) = d(\hat{\beta}) = 0$ if (and only if)) $q \leq \tilde{q}$, and the unique perception-perfect strategy consists in undertaking the investment in the first period. When $q > \tilde{q}$, two cases are possible: either $d(\hat{\beta}) + 1 \leq d(\beta)$, and the investor procrastinates the investment $\bar{L}$, or $d(\beta) = d(\hat{\beta})$, and the investor undertakes the investment $\bar{L}$ within $d(\beta) = d(\hat{\beta})$ periods. In Appendix I prove that lower values of q have a positive effect on the propensity to invest: indeed weaker property rights security on unused assets may imply a shorter delay for the investment and it may lead a present-biased and (partially) naïve investor to undertake the profitable investment $\bar{L}$. That otherwise, with stronger property rights security on unused asset, they would procrastinate.

I summarize as follows the basic intuition for such a result. Suppose that a certain investment on an asset is profitable. When property rights security on unused assets is weaker, delaying the investment means to incur the risk to lose in the future the control of the asset and the possibility to undertake the profitable investment. Therefore property rights insecurity on unused asset reduces the expected returns of delaying without any effect on the profitability of the investment.

4. Concluding Remarks

This paper presents a model of land investment in order to explore the interplay between property rigths and investment incentives when investors' time preferences are described in form of quasi-hyperbolic discounting, whose the standard form of exponential discounting is a particular case. While investors who exponentially discount immediately undertake a profitable investment, individuals characterized by a bias towards present utility may delay or even procrastinate (i.e. continuously delay and never undertake) a profitable investment. For this reason property rights can have an impact on investment incentives not only by affecting the profitability of the investment, but also the *timing* of the investment and this is a totally unexplored field in the literature.

The two major theoretical findings obtained through the analysis of the model we have proposed are the following: 1) the expected ability to get the benefits from a used asset has a non-negative (positive or null) effect on investment incentives of any investor (whatever his time preferences are); 2) the expected ability to undertake a certain investment in future periods (*property rigths security on unused asset*) has a non-positive (negative or null) effect on the investment incentives faced by any investor: while it does not affect investment incentives of time-consistent investors, it strengthens the propensity to delay or procrastinate a profitable investment of time-inconsistent investors.

In this paper we do the crucial assumption of investors credit-constrained, which is a strong assumption which is necessary in order not to make the model too complex but we think that it is a plausible assumption among small and medium landowners in low

income countries. In any case, it seems that also big landowners such as Brazilian landowners, who would have probably the possibility to sell the land or to ask credits to invest on it, tend to be present-biased (indeed they continuously receive occupations of unused land by Sem terra Movement, that only police can avoid). These findings suggest that a more comprehensive view about property rights is necessary in order take in account the most recent indings of behavioural economics.

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Appendix 1 – Proofs of Lemmas and Propositions

Proof of Lemma 1

For any $c, b, \bar{L}, \hat{\beta}, \ \delta, p, q$,

(1) if $\bar{L}$ is not $\hat{\beta}$-worthwhile, then $\widehat{L}_t = 0$ for all $t \geq 2$. Indeed if $\bar{L}$ is not $\hat{\beta}$-worthwhile, no positive investment is $\hat{\beta}$-worthwhile since, $\forall L > 0$, $-cL + \hat{\beta}\frac{\delta p}{1-\delta p}bL < \hat{\beta}[\delta p]^\tau \left[-cL + \frac{\delta p}{1-\delta p}bL\right]$ $\forall \tau \in \{1,2,\dots\}$. The latter inequality implies: $arg \max_{L_t} V^t(c, b, \bar{L}, L_t, h_t, \hat{s}, \hat{\beta}, \delta, p, q) = 0$ for all $t \geq 2$, hence $\widehat{L}_t = 0$ for all $t \geq 2$.

(2) if $\bar{L}$ is $\hat{\beta}$-worthwhile and $d(\hat{\beta}) > 0$, $\exists \tau \in (2,3,\dots, d(\hat{\beta}) + 2)$ such that $\widehat{L}_t = \bar{L}$ if and only if $t \in \{\tau, \tau + d(\hat{\beta}) + 1, \tau + 2(d(\hat{\beta}) + 1), \dots\}$ and $\widehat{L}_t$ otherwise. Indeed if $\bar{L}$ is β-worthwhile and $d(\hat{\beta}) > 0$, given the definition of $d(\hat{\beta})$, for any $d' \in \{1, \dots, d(\hat{\beta})\}$, if $\widehat{L}_t > 0$ and $\widehat{L_{t-d}} = 0$ $\forall d \in \{1, \dots, d' - 1\}$, then $arg \max_{L_t \in A_t} V^{t-d'} = 0$. For $d' = d(\hat{\beta}) + 1$, if $\widehat{L}_t > 0$ and and $\widehat{L_{t-d}} = 0$ $\forall d \in \{1, \dots, d' - 1\}$,, then $arg \max_{L_t \in A_t} V^{t-d'} = \bar{L}$: indeed, if $-c + \beta\frac{\delta p}{1-\delta p}b > 0$, $\bar{L}$ is the unique $arg \max_{L_t \in A_t} V^{t-d'}$. If instead $-c + \beta\frac{\delta p}{1-\delta p}b = 0$, any $L > 0$ is $arg \max_{L_t \in A_t} V^{t-d'}$ but *assumption Q* holds: hence $\hat{s}$ must have $\widehat{L}_t = \bar{L}$ every $d(\hat{\beta} + 1)$ periods and $\widehat{L}_t = 0$ otherwise. This condition can be satisfied only if $min\{t \in (2,3,\dots)/\widehat{L}_t = \bar{L}\} \in \{2, \dots, d(L/\hat{\beta}) + 2\}$.

(3) If $\bar{L}$ is $\hat{\beta}$-worthwhile and $d(\hat{\beta}) = 0$, it is straighforward that the unique dynamically consistent beliefs is $(\widehat{L_2}, \widehat{L_3}, \dots) = (\bar{L}, 0, 0, \dots)$.

Proof of Lemma 2

For any $c, b, \bar{L}, \beta, \hat{\beta}, \ \delta, p, q$,

(1) if $\bar{L}$ is not β-worthwhile, then $S^{pp} = \{s^0\}$. Indeed, if $\bar{L}$ is not β-worthwhile, for any $L > 0$, for any $\hat{s}(\hat{\beta}, \delta)$ and for all t, $V^t(L, \hat{s}(\hat{\beta}, \delta), \beta, \delta) < 0 \leq V^t(0, \hat{s}(\hat{\beta}, \delta), \beta, \delta)$, hence $S^{pp} = \{s^0\}$.

(2) if $\bar{L}$ is β-worthwhile and $d(\hat{\beta}) + 1 \leq d(\beta)$, $S^{pp} = \{s^0\}$ (*procrastination*). Indeed, suppose that $\bar{L}$ is β-worthwhile and $d(\hat{\beta}) + 1 \leq d(\beta)$. $\bar{L}$ is also $\hat{\beta}$-worthwhile, so that for any $\hat{s}(\hat{\beta}, \delta)$ and for all t, $V^t(0, \hat{s}(\hat{\beta}, \delta), \beta, \delta) = \beta[\delta q]^d \left[-c\bar{L} + \frac{\delta p}{1-\delta p}b\bar{L}\right]$ for some $d \in \{1,2,\dots, d(\hat{\beta}) + 1\}$. This implies that , $V^t(0, \hat{s}(\hat{\beta}, \delta), \beta, \delta) \geq \beta[\delta q]^{d(\hat{\beta})+1}\left[-c\bar{L} + \frac{\delta p}{1-\delta p}b\bar{L}\right]$ for all t. Since $V^t(\bar{L}, \hat{s}(\hat{\beta}, \delta), \beta, \delta) = -cL + \frac{\delta p}{1-\delta p}b$, a positive investment L can be $argmax\, V^t(L, \hat{s}(\hat{\beta}, \delta), \beta, \delta)$ only if $-cL + \beta\frac{\delta p}{1-\delta p}bL \geq \beta[\delta q]^{d(\hat{\beta})+1}\left[-cL + \frac{\delta p}{1-\delta p}bL\right]$. But since the definition of $d(\beta)$ implies that, for all $L > 0$, $-cL + \beta\frac{\delta p}{1-\delta p}bL < \beta[\delta q]^d\left[-cL + \frac{\delta p}{1-\delta p}bL\right]$ for all $d \leq d(\beta)$, hence $d(\hat{\beta}) + 1 \leq d(\beta)$ implies $-cL + \beta\frac{\delta p}{1-\delta p}bL < \beta[\delta q]^{d(\hat{\beta})+1}\left[-cL + \frac{\delta p}{1-\delta p}bL\right]$ for all $L > 0$. Therefore, if L is β-worthwhile but $d(\hat{\beta}) + 1 \leq d(\beta)$, for any $\hat{s}(\hat{\beta}, \delta)$, $S^{pp} = \{s^0\}$ (*procrastination*).

(3) If $\bar{L}$ is β-worthwhile and $d(\beta) = d(\hat{\beta}) > 0$, $s^0 \notin S^{pp}$ and any perception perfect strategy $s^{pp} \in S^{pp}$ must satisfy: $L^{pp} = \bar{L}$ for $t \in \{\tau(s^{pp}), \tau(s^{pp}) + d(\beta) + 1, \tau(s^{pp}) + 2(d(\beta) + 1), \dots\}$ and $L_t^{pp} = 0$ otherwise, where either $\tau(s^{pp}) = 1$ or $\tau(s^{pp}) = \tau(\hat{s})$. Indeed suppose that L is β-worthwhile and $d(\beta) = d(\hat{\beta}) > 0$: L is $\hat{\beta}$-worthwhile, so $\hat{s}$ must have $\widehat{L}_t = \bar{L}$ every and $d(\hat{\beta}) + 1$ periods and $\widehat{L}_t = 0$ otherwise. The definition of $d(\beta)$, the hypothesis of constant returns to scale and *assumption Q* imply that $s^0 \notin S^{pp}$ and that for any $\hat{s}(\hat{\beta}, \delta)$, the associated perception perfect

strategy must satisfy: $L_t^{pp} = \bar{L}$ if and only if $min\{d \in \{1,2,\dots\}/\hat{L}_{t+d}(\beta,\delta) = \bar{L}\} = d(\beta) + 1 = d(\hat{\beta}) + 1$ and $L_t^{pp} = 0$ otherwise. Hence $\tau(s^{pp}) = min\{t \in \{1,2,\dots\}/min\{d \in \{1,2,\dots\}/\hat{L}_{t+d}(\beta,\delta) = \bar{L}\} = d(\hat{\beta}) + 1\}$. This means that $\tau(s^{pp}) = 1$ when $\tau(\hat{s}) = d(\hat{\beta}) + 2$ and $\tau(s^{pp}) = \tau(\hat{s})$ otherwise.

(4) If $\bar{L}$ is β-worthwhile and $d(\beta) = d(\hat{\beta}) = 0$, the unique perception perfect strategy is $s^{pp} = (\bar{L}, 0, 0, \dots)$. Indeed if $\bar{L}$ is β-worthwhile and $d(\beta) = d(\hat{\beta}) = 0$, the unique dynamically consistent belief is $(\hat{L}_2, \hat{L}_3, \dots) = (0, 0, \dots)$. Since a one-period delay is not profitable, it is straighforward that the unique perception perfect strategy is $s^{pp} = (\bar{L}, 0, 0, \dots)$.

Proof of Proposition 1

Consider $0 < \delta < 1$ and $\beta = \hat{\beta} = 1$.

(1) If $-c + \frac{\delta p}{1-\delta p} b < 0$, Lemma 2 proves that $S^{pp} = \{s^0\}$. Suppose that $-c + \frac{\delta p}{1-\delta p} b \geq 0$. Since $d(\beta = 1) = 0$, the unique perception perfect strategy is $s^{pp} = (\bar{L}, 0, 0, \dots)$.

(2) Given b, c, δ, if $-c + \frac{\delta}{1-\delta} b < 0$, then $-c + \frac{\delta p}{1-\delta p} b < 0$ for any $0 < p \leq 1$, hence $S^{pp} = \{s^0\}$. If instead $-c + \frac{\delta}{1-\delta} b \geq 0$, let $0 < \tilde{p} = \frac{c}{\delta(b+c)} < 1$. If $p \geq \tilde{p}$, $-c + \frac{\delta p}{1-\delta p} b \geq 0$ and the unique perception perfect strategy is $s^{pp} = (\bar{L}, 0, 0, \dots)$. If instead $p < \tilde{p}$, $-c + \frac{\delta p}{1-\delta p} b < 0$ and $S^{pp} = \{s^0\}$.

(3) It is straighforward since, as it is evident in (1), the parameter q does not affect perception perfect strategies.

Proof of Proposition 2

If $0 < \delta < 1$ and $\beta < 1$:

(1) if $-c + \frac{\delta p}{1-\delta p} b < 0$, $S^{pp} = \{s^0\}$; if instead $-c + \frac{\delta p}{1-\delta p} b \geq 0$, when $d(\hat{\beta}) + 1 \leq d(\beta)$, $S^{pp} = \{s^0\}$ (*procrastination*); when $d(\hat{\beta}) = d(\beta)$, $S^{pp} \neq \{s^0\}$ and in particular if $d(\hat{\beta}) = d(\beta) = 0$, $s^{pp} = (\bar{L}, 0, 0, \dots)$ whereas if $d(\hat{\beta}) = d(\beta) > 0$, there exist multiple perception perfect strategies (see Lemma 2).

(2) Given b, c, δ, β, if $-c + \frac{\delta}{1-\delta} b < 0$, it is straighforward that, for any $0 < p \leq 1$, $-c + \frac{\delta p}{1-\delta p} b < 0$. Hence, for any $0 < q \leq 1$ and any $\hat{\beta}$, $S^{pp} = \{s^0\}$. If instead $-c + \frac{\delta}{1-\delta} b \geq 0$, let $0 < \tilde{p} = \frac{c}{\delta(b+c)} \leq 1$. If $p < \tilde{p}$, $-c + \frac{\delta p}{1-\delta p} b < 0$ and, for any $0 < q \leq 1$ and any $\hat{\beta}$, $S^{pp} = \{s^0\}$. If instead $p \geq \tilde{p}$, $-c + \frac{\delta p}{1-\delta p} b \geq 0$ and $S^{pp} \neq \{s^0\}$ if and only if $d(\beta) = d(\hat{\beta})$.

(3) By definition $d(\beta) = max\left\{d \in \{0,1,\dots\}/-c + \beta \frac{\delta p}{1-\delta p} b < \beta(\delta q)^d \left[-c + \frac{\delta p}{1-\delta p} b\right]\right\}$. Hence $d(\beta) = 0$ if and only if $-c + \beta \frac{\delta p}{1-\delta p} b > \beta \delta q \left[-c + \frac{\delta p}{1-\delta p} b\right]$, i.e. if and only if $q < \frac{-c + \delta pc + \beta \delta pb}{\beta \delta(-c + \delta pc + \delta pb)} \equiv \tilde{q}$. We know from Lemma 2 that if $-c + \frac{\delta p}{1-\delta p} b \geq 0$ and $d(\beta) = 0$, the unique perception perfect strategy is $s^{pp} = (\bar{L}, 0, 0, \dots)$. If instead $q > \tilde{q}$, $d(\beta) > 0$ and Lemma 2 suggests that there are $d(\beta) + 1$ perception perfect strategies, where any perception perfect strategy $s^{pp} \in S^{pp}$ must satisfy: $L_t^{pp} = \bar{L}$ for $t \in \{\tau(s^{pp}), \tau(s^{pp}) + d(\beta) + 1, \tau(s^{pp}) + 2(d(\beta) + 1), \dots\}$ and $L_t^{pp} = 0$ otherwise, where $\tau(s^{pp}) \leq \tau(\hat{s}) \leq d(\beta) + 1$. Given the definition of $d(\beta)$, it is straighforward that $d(\beta)$ is non decreasing in q and thus also in $\tau(s^{pp})$.

Suppose now $-c + \beta \frac{\delta p}{1-\delta p} b \geq 0$ and $\beta < \hat{\beta} \leq 1$ (naif or partially naif investor). Given b, c, δ, p, β, let $\tilde{q} = \frac{-c+\delta pc+\beta\delta pb}{\beta\delta(-c+\delta pc+\delta pb)}$. For any β and $\hat{\beta}$ such that $\beta < \hat{\beta} \leq 1$, $d(\beta) = 0$ if and only if $q \leq \tilde{q}$, i.e. $s^{pp} = (\bar{L}, 0,0, \dots)$ if and only if $q \leq \tilde{q}$. If instead $q > \tilde{q}$, $d(\beta) > 0$ and if $d(\hat{\beta}) + 1 \leq d(\beta)$, $S^{pp} = \{s^0\}$ (*procrastination*), whereas if $d(\hat{\beta}) = d(\beta)$, any $s^{pp} \in S^{pp}$ must satisfy: $L_t^{pp} = \bar{L}$ for $t \in \{\tau(s^{pp}), \tau(s^{pp}) + d(\beta) + 1, \tau(s^{pp}) + 2(d(\beta) + 1), \dots\}$ and $L_t^{pp} = 0$ otherwise, where, given the definition of $d(\beta)$, $d(\beta)$ is non decreasing in q and also in $\tau(s^{pp})$. Suppose now that there exists $\ddot{q} > \tilde{q}$ such that $S^{pp} \neq \{s^0\}$, meaning that $d(\beta) = d(\hat{\beta})$. Hence $\forall q \in (\tilde{q}, \ddot{q})$, $S^{pp} \neq \{s^0\}$. Indeed by definition: $d(\beta) = max\left\{ d \in \{0,1, \dots\} / (\delta q)^d > \frac{\frac{1}{\beta}(-c+\delta pc)+\delta pb}{-c+\delta pc+\delta pb} \right\}$ and $d(\hat{\beta}) = max\left\{ d \in \{0,1, \dots\} / (\delta q)^d > \frac{\frac{1}{\hat{\beta}}(-c+\delta pc)+\delta pb}{-c+\delta pc+\delta pb} \right\}$. Let call $\frac{\frac{1}{\beta}(-c+\delta pc)+\delta pb}{-c+\delta pc+\delta pb} \equiv A$ and $\frac{\frac{1}{\hat{\beta}}(-c+\delta pc)+\delta pb}{-c+\delta pc+\delta pb} \equiv B$, where $B \geq A$. If, for $q = \ddot{q}$, $d(\beta) = d(\hat{\beta}) = \tilde{d}$, this means that $(\delta\ddot{q})^{\tilde{d}} > B \geq A$. Consider now any $q \in (\tilde{q}, \ddot{q})$. Since $d(\beta)$ is non decreasing in q, it must be that $d(\beta) \leq \tilde{d}$ and $d(\hat{\beta}) \leq \tilde{d}$. If $d(\beta) = \tilde{d}$, also $d(\hat{\beta}) = \tilde{d}$ and $S^{pp} \neq \{s^0\}$. Consider instead $d(\beta) = d' < \tilde{d}$; this means that $(\delta\ddot{q})^{\tilde{d}} \leq (\delta\ddot{q})^{d'+1} \leq A$. Since $B \geq A$, it must be that $(\delta\ddot{q})^{\tilde{d}} \leq (\delta\ddot{q})^{d'+1} \leq B$ and hence $d(\hat{\beta}) = d(\beta)$.

Suppose now that there esists $\bar{q} > \tilde{q}$ such that $S^{pp} = \{s^0\}$ (*procrastination*). Hence there exists q^*, where $\tilde{q} \leq q^* < \bar{q}$ such that $S^{pp} \neq \{s^0\}$ for any $q \leq q^*$. Indeed $S^{pp} = \{s^0\}$ means that $d(\hat{\beta}) + 1 \leq d(\beta)$. Since both $d(\beta)$ and $d(\hat{\beta})$ are non decreasing in q, it may be (depending on the parameters) that there exists $q^* > \tilde{q}$ such that $d(\beta) = d(\hat{\beta})$, which would imply that $S^{pp} \neq \{s^0\}$ for any $q \leq q^*$, where $q^* > \tilde{q}$. If instead for any $q^* > \tilde{q}$ I have that $d(\hat{\beta}) + 1 \leq d(\beta)$, we know that for any $q \leq \tilde{q}$ the unique perception perfect strategy is $s^{pp} = (\bar{L}, 0,0, \dots)$.